SOLUTION TO ENVIRONMENTAL/ECONOMIC DISPATCH PROBLEM BY USING FIRST ORDER GRADIENT METHOD

Celal YAŞAR[1] Salih FADIL[2]

e-mail: cyasar@dumlupinar.edu.tr e-mail: sfadil@ogu.edu.trr

[1] *Dumlupınar University, Department of Electrical Engineering, 43100, Kütahya, TURKEY*

[2] *Eskişehir Osmangazi University, Department of Electrical Engineering, 26480, Eskişehir, TURKEY*

Key Words: Environmental/economic dispatch, Gradient method, Reference bus penalty factors.

ABSTRACT

A solution technique based on first order gradient method to find Pareto-optimal solutions of an environmental/economic dispatch problem of a lossy electric power system is given. The transmission losses are incorporated into the solution process via the reference bus penalty factors. These reference bus penalty factors are obtained from Jacobian matrix that is calculated at the end of Newton-Raphson iterations of the load flow calculations. The solution technique gives the minimum total cost rate (total thermal cost rate plus total emission cost rate) under the electric constraints. The solution technique is tested on an electric power system and the obtained results are given.

I. INTRODUCTION

In a traditional economic power dispatch problem, the units' active generation powers in a considered electric power system is tried to be calculated so that the total thermal cost rate becomes minimum. The active generation powers should also satisfy the electric constraints in the considered electric power system [1-3].

Nowadays, environmental pollution created by some type of thermal units has become an important issue. Thermal units, which burn fossil fuel, emit carbon dioxide, sulfur dioxide, nitrogen oxide and ash. Increase of those emissions in large amounts can result in some deadly environmental effects such as global warming [4, 5].

The solution obtained from a traditional economic dispatch can not be taken the best one since the environmental criteria are not taken into consideration in a traditional economic dispatch calculation. In order to have a cleaner environment, the amount of emission, produced by the thermal units, must be decreased. This can be done in different ways such as using fuels with lower sulfur content, adding units to the generation plants that decrease the carbon dioxide, sulfur dioxide, nitrogen oxide and ash emission or using new dispatch techniques that consider the above emissions. The main idea in the new dispatch techniques is based on employing more generation units that give less emission in order to reduce the amount of total emission [2-4].

In the literature, a multi-objective economic dispatch problem was solved by using various solution methods. Some of these methods use multi-objective genetic (or modified genetic) algorithm [1,7-9], fuzzy linear programming [2,10], hierarchical system approach [3], fuzzified multi-objective particle swarm optimization algorithm [4], fast Newton-Raphson algorithm [6], linear programming [11]. In reference [5], a summary of environmental/economic dispatch algorithms is also given.

In some optimization problems, there may be more than one objective function to be optimized. None of these objectives can be comparable with the others. Generally, in that type of optimization problems, there is no a unique solution, but a set of solutions. If all objectives are taken into consideration, none of the solution in the solution set can be taken as the best one. These types of solutions are named as Pareto-optimal solutions [12].

In this paper, a solution to a lossy environmental/economic active generation dispatch problem with two objective functions is given. The solution technique is based on first order gradient method. It starts the solution process with a selected feasible solution and reaches the optimal solution going from one feasible solution to another by making some decrease in the total cost rate. The solution process (iteration) continues until the decrease in the total cost rate is less than a predetermined tolerance value [13].

II. PROBLEM FORMULATION

The solution to an environmental/economic dispatch problem gives active power generations for all generation units that minimize the total cost rate, which is the summation of the total thermal cost rate and the total emission cost rate. The solution also satisfies all possible electric constraints.

The thermal cost rate (cost per hour) functions of the thermal units in the considered electric power system are taken as follows:

$$F_n(P_{G,n}) = a_n + b_n P_{G,n} + c_n P_{G,n}^2 \,,\; n = ref, n \in N_G \,,(R/h)^1 \tag{1}$$

The emission rate functions of the thermal units in the considered electric power system are also taken as follows:

$$E_n(P_{G,n}) = d_n + e_n P_{G,n} + f_n P_{G,n}^2 \,,\; n = ref, n \in N_G \,,\; (ton/h) \tag{2}$$

The value of $P_{G,n}$ in (1) and (2) represents the active power generation (as *MW*) of the thermal unit that is connected to bus *n* in the considered power system. N_G and ref in (1) and (2) also denote for the set containing all thermal units (*excluding* the one connected to the reference bus) and the reference bus in the system, respectively.

The active power balance constraint is given as follows:

$$\sum_{n \in N_G} P_{G,n} + P_{G,ref} - P_{load} - P_{loss} = 0 \,, \tag{3}$$

where P_{load} and P_{loss} stand for the total active load and loss in the system, respectively.

The active power generation limits of the thermal units are given below:

$$P_{G,n}^{min} \le P_{G,n} \le P_{G,n}^{max} \,,\; n = ref, n \in N_G \tag{4}$$

[1] R stands for a fictitious monetary unit.

The cost rate function of the *n*th thermal unit for the environmental/economic dispatch problem is chosen as follows:

$$T_n(P_{G,n}) = wF_n(P_{G,n}) + (1-w)\,\gamma_n E_n(P_{G,n}) \,,\; n = ref,\; n \in N_G \,,\; (R/h) \tag{5}$$

where γ_n and w represent the emission cost (as *R/ton*) of the *n*th thermal unit and a weight factor $(0 \le w \le 1)$, respectively. If the value of w is taken as equal to one, only the total thermal cost rate is considered, but if the value of w is taken as equal to zero, only the total emission cost rate is considered in the solution process [2,3,6]. The total cost rate of the considered electric power system *that is to be minimized* is given as:

$$T_{total} = \sum_{n \in N_G} T_n(P_{G,n}) + T_{ref}\,(P_{G,ref}), \quad (R/h) \tag{6}$$

III. THE SOLUTION METHOD

From Equation (6), by retaining only the first order derivatives, the change in the total cost rate may be expressed as follows:

$$\Delta T_{total} = \sum_{n \in N_G} \frac{dT_n(P_{G,n})}{dP_{G,n}} \Delta P_{G,n} + \frac{dT_{ref}(P_{G,ref})}{dP_{G,ref}} \Delta P_{G,ref} \tag{7}$$

where $\Delta P_{G,n}$ and $\Delta P_{G,ref}$ denote for the changes in active generation powers of the units connected to bus *n* and the reference bus, respectively.

Similarly, the change for Equation (3) can be expressed as follows:

$$\Delta P_{G,ref} = \Delta P_{loss} - \sum_{n \in N_G} \Delta P_{G,n} \tag{8}$$

Since P_{load} is a constant, it does not appear in the change expression.

The change in the total loss becomes:

$$\Delta P_{loss} = \sum_{n \in N_G} \frac{\partial P_{loss}}{\partial P_{G,n}} \Delta P_{G,n} + \frac{\partial P_{loss}}{\partial P_{G,ref}} \Delta P_{G,ref} \tag{9}$$

If Equation (9) is substituted in Equation (8) and the necessary rearrangements are made, the change in $P_{G,ref}$ becomes:

$$\Delta P_{G,ref} = -\sum_{n \in N_G} \beta_{G,n} \; \Delta P_{G,n} \tag{10}$$

Since the reference bus penalty factors are used in the solution technique, $\partial P_{loss} / \partial P_{G,ref} = 0$ is taken in the derivation of Equation (10). The value of $\beta_{G,n}$ in Equation (10) is defined as follows:

$$\beta_{G,n} = 1 - \frac{\partial P_{loss}}{\partial P_{G,n}} \tag{11}$$

It is the inverse of the penalty factor for the *n*th thermal unit. It is also calculated from Jacobian matrix that is found in the Newton-Raphson load flow calculation (*please see the appendix section for the derivation of the inverse of penalty factors*) [13].

Substituting $\Delta P_{G,ref}$ into Equation (7), a new expression for ΔT_{total} is obtained.

$$\Delta T_{total} = \sum_{n \in N_G} \left[\frac{dT_n(P_{G,n})}{dP_{G,n}} - \beta_{G,n} \frac{dT_{ref}(P_{G,ref})}{dP_{G,ref}} \right] \Delta P_{G,n} \tag{12}$$

A new total cost rate value can be calculated from the previous total cost rate, $T_{total}^{(old)}$, and the change in the previous total cost rate, $\Delta T_{total}^{(old)}$, according to:

$$T_{total}^{(new)} = T_{total}^{(old)} + \Delta T_{total}^{(old)} \tag{13}$$

The new total cost rate must be smaller than the previous one. Therefore, the change in total cost rate, $\Delta T_{total}^{(old)}$, is tried to be made as more negative as possible in the given solution technique. The new active generation, which makes the change in the total cost rate negative, is determined. With this new active generation, a new load flow calculation is made. This process continues until the stopping criterion is satisfied:

$$(T_{total}^{(g)} - T_{total}^{(g+1)}) \le TOL_{\Delta T_{total}} \tag{14}$$

where *g* and $TOL_{\Delta T_{total}}$ represent an iteration number and a selected tolerance value for the total cost rate decrease, respectively.

IV. SOLUTION ALGORITM

Step-1: The iteration number is taken as $g = 0$. The initial active generations are selected in such a way that $P_{G,n}^{(g)}$ values satisfies the constraints given in Equation (4) and $\sum_{n \in N_G} P_{G,n} \le P_{load}$ (the thermal unit connected to the reference bus should work in generation mode), (*selection of an initial feasible solution*). A load flow calculation is carried out with the selected initial active generations. After that, $P_{G,ref}^{(g)}$, $\beta_{G,n}^{(g)}$, $n \in N_G$, $F_{total}^{(g)}$, $E_{total}^{(g)}$, $T_{total}^{(g)}$ are calculated. The values of total thermal cost rate and total emission rate ($F_{total}^{(g)}$ and $E_{total}^{(g)}$) are defined as follows:

$$F_{total}^{(g)} = \sum_{n = ref, n \in N_G} F_n(P_{G,n}^{(g)}), \quad (R/h) \tag{15}$$

$$E_{total}^{(g)} = \sum_{n = ref, n \in N_G} E_n(P_{G,n}^{(g)}), \quad (ton/h) \tag{16}$$

Step-2: The coefficients of $\Delta P_{G,n}^{(g)}, n \in N_G$ in Equation (12), whose total number is equal to $S\{N_G\}$, are calculated. The number of elements (thermal units, *excluding* the one connected to reference bus) in set N_G is represented as $S\{N_G\}$.

Step-3: If the coefficient of $\Delta P_{G,n}^{(g)}$ is positive, $\Delta P_{G,n}^{(g)}$ is taken as negative and also selected according to the following expression:

$$\left| \Delta P_{G,n}^{(g)} \right| = \alpha_G (P_{G,n}^{(g)} - P_{G,n}^{min}), \; 0 < \alpha_G \le 1 \tag{17}$$

Negative $\Delta P_{G,n}^{(g)}$ is balanced with an opposite and equal change (increase) on the active generation of the unit connected to the reference bus. At the same time, $\Delta P_{G,n}^{(g)} < 0$ causes some change (increase or decrease) on the transmission loss in the considered power system. This transmission loss change causes an equal change (increase or decrease) on the active generation of the unit connected to the reference bus. Therefore, $\Delta P_{G,n}^{(g)} < 0$ should satisfy the inequality given below:

$$\left| \Delta P_{G,n}^{(g)} \right| < (P_{G,ref}^{max} - P_{G,ref}^{(g)}) \tag{18}$$

If the coefficient of $\Delta P_{G,n}^{(g)}$ is negative, $\Delta P_{G,n}^{(g)}$ is taken as positive and also selected according to the expressions,

$$\Delta P_{G,n}^{(g)} = \alpha_G (P_{G,n}^{max} - P_{G,n}^{(g)}) \tag{19}$$

$$\Delta P_{G,n}^{(g)} < (P_{G,ref}^{(g)} - P_{G,ref}^{min}) \tag{20}$$

α_G in Equations (17) and (19) is a coefficient between 0 and 1 (inclusive).

Step-4: The changes in generations, $\Delta P_{G,n}^{(g)}, n \in N_G$, which are selected in *step-3,* their corresponding coefficients, which are calculated in *step-2,* are multiplied. Among those $S\{N_G\}$ products, the most negative valued one is selected. Let us assume that it contains $\Delta P_{G,a}^{(g)}$. In that case, the *a*th thermal unit's new active power generation is calculated as follows:

$$P_{G,a}^{(g+1)} = P_{G,a}^{(g)} + \Delta P_{G,a}^{(g)} \tag{21}$$

With the new $P_{G,a}^{(g+1)}$ value, a power flow calculation is carried out and, the new values of $P_{G,ref}^{(g+1)}$, $\beta_{G,n}^{(g+1)}$, $\forall n \in N_G$, $F_{total}^{(g+1)}$, $E_{total}^{(g+1)}$, $T_{total}^{(g+1)}$ are calculated.

Step-5: The stopping criterion given in Equation (14) is checked. If it is satisfied, the solution process is stopped and the solution is obtained. If the sopping criterion is not satisfied, the iteration number is incremented by one ($g = g+1$) and the solution process proceeds by returning to *step-2*.

V. EXAMPLE

The solution technique is applied to the electric power system with 10 buses, which is given in references [10] and [11]. The emission cost and the tolerance value in Equation (14) are taken as $\gamma_n = 1000\ R/ton, n = 1,7,8,9,10$ and $TOL_{\Delta T_{total}} = 1.0 \times 10^{-4}$ *R*, respectively.

The effect of the weight coefficient, *w*, on the total thermal cost rate and the total emission rate is clearly seen in Figure 1.

When the value of *w* in Equation (5) is taken as equal to one (the emission cost rates are ignored), the total thermal cost rate is found as 165.122 *R/h.* The emission rate in this case becomes 171.071 *kg/h*. Once the value of w is incremented from 0.0 to 1.0 by 0.1, the total thermal cost rate decreases, whereas the total emission rate increases. This situation is clearly seen in Figure 1. When the value of w is taken as equal to zero (the thermal cost rates are ignored), the total thermal cost rate and the total emission rate become 166.523 *R/h* and 156.338 *kg/h*, respectively. As the value of w changes from 0.0 to 1.0, the changes in the total thermal cost and in the total emission rate are obtained as 1.401 *R/h* (decrease) and 15.363 *kg/h* (increase), respectively

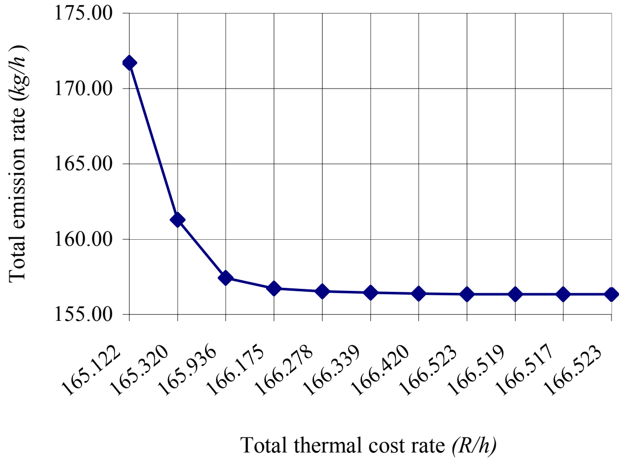

Figure 1. The effect of *w* value on the total thermal cost and the total emission rate.

VI. CONCLUSION

A solution technique, based on first order gradient method, for environmental/economic dispatch problem of a lossy electric power system is given. The solution technique is tested on an example electric power system with 10 buses. For each value of *w,* it starts the solution process with a selected feasible solution and reaches the optimal solution going from one feasible solution to another by making some decrease in the total cost rate. In the solution process, the value of *w* is incremented from 0.0 to 1.0 by 0.1. The total thermal cost rate and total emission rate values at solution points for each values of *w* are given graphically.

All kinds of constraints in the considered problem can be controlled very easily by the given solution technique. Since it starts with a feasible solution and reaches the optimal solution going from one feasible solution to another by making some decrease in the total cost rate, all intermediate solutions are also feasible and can be applied to the power system under consideration.

One of the disadvantages of the given solution technique is that there is no clear rule in selection of the amount of changes (selection of α_G) in iterations. Generally, the changes are taken relatively large at the beginning, and decreased with certain percentage as the iteration proceeds. Also, the active generation of only one unit is changed in any iteration. Those disadvantages can increase the number of iterations done in the solution technique.

At the end of a previous iteration, the thermal unit connected to the reference bus may already hit one of its generation limits and the selection of an active generation change in the current iteration may necessitate the active generation of the thermal unit connected to the reference bus to go beyond its limit. If this is the case, the solution procedure can not go further. In that case, another bus, to which a thermal unit (whose active generation capacity is higher than the previous one) is connected, should be taken as the new reference bus. That also requires reinitialization of the solution procedure.

APPENDIX

$\Delta P_{G,ref}$ can be written as follows:

$$\Delta P_{G,ref} = \sum_{n \in N_G} \frac{\partial P_{G,ref}}{\partial \delta_n} \Delta \delta_n + \sum_{n \in N_G} \frac{\partial P_{G,ref}}{\partial |U_n|} \Delta |U_n| \quad \text{(A1)}$$

where $\Delta \delta_n$ and $\Delta |U_n|$ denote for the changes in phase angle and magnitude of voltage of bus *n*, respectively. By using the chain rule in derivative, Equation (A1) can be given as follows:

$$\Delta P_{G,ref} = \sum_{n \in N_G} \frac{\partial P_{G,ref}}{\partial \delta_n} \frac{\partial \delta_n}{\partial P_{G,n}} \Delta P_{G,n} + \sum_{n \in N_G} \frac{\partial P_{G,ref}}{\partial |U_n|} \frac{\partial |U_n|}{\partial P_{G,n}} \Delta P_{G,n} \quad \text{(A2)}$$

$$\Delta P_{G,ref} = \sum_{n \in N_G} \frac{\partial P_{G,ref}}{\partial \delta_n} \frac{\partial \delta_n}{\partial Q_{G,n}} \Delta Q_{G,n} + \sum_{n \in N_G} \frac{\partial P_{G,ref}}{\partial |U_n|} \frac{\partial |U_n|}{\partial Q_{G,n}} \Delta Q_{G,n} \quad \text{(A3)}$$

$Q_{G,n}$ in Equation (A3) represents the reactive power generation of the thermal unit connected to bus *n*. The terms $\frac{\partial \delta_n}{\partial P_{G,n}}$, $\frac{\partial |U_n|}{\partial P_{G,n}}$, $\frac{\partial \delta_n}{\partial Q_{G,n}}$, $\frac{\partial |U_n|}{\partial Q_{G,n}}$ seen in Equations (A2) and (A3) are the terms of inverse Jacobian matrix. By considering (A2) and (A3), the following matrix equation can be written.

$$\left[\frac{\partial P_{G,ref}}{\partial P_{G,1}} .. \frac{\partial P_{G,ref}}{\partial P_{G,n}}, \frac{\partial P_{G,ref}}{\partial Q_{G,1}} .. \frac{\partial P_{G,ref}}{\partial Q_{G,n}} \right] =$$

$$\left[\frac{\partial P_{G,ref}}{\partial \delta_1} .. \frac{\partial P_{G,ref}}{\partial \delta_n}, \frac{\partial P_{G,ref}}{\partial |U_1|} .. \frac{\partial P_{G,ref}}{\partial |U_n|} \right] [\boldsymbol{J}]^{-1} \quad \text{(A4)}$$

$[\boldsymbol{J}]^{-1}$ in Equation (A4) denotes for the inverse Jacobian matrix at the solution point. Instead of taking a matrix inverse, solution of a linear equation set is preferred. Because of it, the following linear equation set is used to calculate the negative of inverse penalty factors [13].

$$[\boldsymbol{J}]^T \left[\frac{\partial P_{G,ref}}{\partial P_{G,1}} .. \frac{\partial P_{G,ref}}{\partial P_{G,n}}, \frac{\partial P_{G,ref}}{\partial Q_{G,1}} .. \frac{\partial P_{G,ref}}{\partial Q_{G,n}} \right]^T =$$

$$\left[\frac{\partial P_{G,ref}}{\partial \delta_1} .. \frac{\partial P_{G,ref}}{\partial \delta_n}, \frac{\partial P_{G,ref}}{\partial |U_1|} .. \frac{\partial P_{G,ref}}{\partial |U_n|} \right]^T \quad \text{(A5)}$$

REFERENCES

1. Zhang P.X., Zhao B., Cao Y.J., Cheng S.J., "A Novel Multi-Objective Genetic Algorithm For Economic Power Dispatch", *Universities Power Engineering Conference, 2004. UPEC 2004, 39th International*, Vol. 1, pp.422-426, 6-8 September 2004.
2. Yang H.T., Huang C.M., Lee H.M., Huang C.L., "Multi-objective Power Dispatch Using Fuzzy Linear Programming", *IEEE Transactions on Energy Conversion,* Vol. 12, No.1, pp. 86-93, 1997.
3. Hu Y.L., Wee W.G., "A Hierarchical System for Economic Dispatch with Environmental Constraints", *IEEE Transactions on Power Systems*, Vol. 9, No.2, pp.1076-1082, 1994.
4. Wang L., Singh C., "Environmental/Economic Power Dispatch Using a Fuzzified Multi-Objective Particle Swarm Optimization Algorithm", *Electric Power Systems Research*, article in press.
5. Talaq J.H., El-Hawary F. and El-Hawary M.E., "A Summary of Environmental/Economic Dispatch Algorithms", *IEEE Transactions on Power Systems*, Vol. 9, No.3, pp. 1508-1516, 1994.
6. Chen J.F., Chen S.D., "Multiobjective Power Dispatch with line Flow Constraints Using The Fast Newton-Raphson Method", *IEEE Transactions on Energy Conversion*, Vol. 12, No.1, pp. 86-93, 1997.
7. [Yalçınöz T., Altun H., "Environmentally Constrained Economic Dispatch via A Genetic Algorithm with Arithmetic Crossover", *Africon Conference in Africa*, *IEEE AFRICON 6th*, Vol. 2, pp.923-928, 2-4 Oct. 2002.
8. Abido M.A., "Environmental/Economic Power Dispatch Using Multi-objective Evolutionary Algorithms", *IEEE Transactions on Power Systems*, Vol. 18, No.4, pp.1529-1537, 2003.
9. Ah King R.T.F., Rughooputh H.C.S., "Elitist Multiobjective Evolutionary Algorithm For Environmental/Economic Dispatch", *IEEE Evolutionary Computation, CEC'03, The 2003 Congress on*, Vol. 2, pp.1108-1114, 8-12 Dec. 2003.
10. Yang H.T., Huang C.M., Lee H.M., Huang C.L.,"Multiobjective Power Dispatch Using Fuzzy Linear Programming", *IEEE Energy Management and Power Delivery, proceedings of EMPD'95, 1995 International Conference on*, Vol. 2, pp.738-743,21-23 Nov. 1995.
11. Farag A., Al-Baiyat S., Cheng T.C., "Economic Load Dispatch Multi-objective Optimization Procedures Using Linear Programming Techniques", *IEEE Transactions on Power Systems,* Vol. 10, No.2, pp.731-738, 1995.
12. Zitzler E., Thiele L., "Multi-objective Optimization Using Evolutionary Algorithms- A Comparative Case Study", *Parallel Problem Solving From Nature-PPSN V, Amsterdam, The Netherlands, Springer-Verlag*, September 1998, pp.292-301.
13. Wood A. J., Wollenberg B. F., Power Generation Operation and Control, New York-John & Wiley, 1996.