OPTIMAL DESIGN OF MINIMUM PHASE DIGITAL FIR FILTERS BY USING GENETIC ALGORITHM

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ABSTRACT

Genetic Algorithm (GA) based design techniques are widely proposed for Finite Impuls Response (FIR) filters. In this work, an effective design method for minimum phase digital FIR filters using GA is presented. While obtaining the optimal magnitude response, to optimize the passband and the stopband responses the Mean Squared Error function is used and to optimize the transition band response the Mean Absolute Error is used.

I. INTRODUCTION

A digital FIR filter is characterized by the following transfer function,

$$H(z) = \sum_{n=0}^{N} a_n z^{-n} \qquad (1)$$

In the above expression, N is the order of the filter and a_n represent the filter coefficients to be determined in the design process. Designing the FIR filters as minumum phase provides some important advantages. Minimum phase filters have two main advantages: reduced filter length and minimum group delay. Minimum phase filters can simultaneously meet delay and magnitude response constraints yet generally require fewer computations and less memory than linear phase[1].

Recently, GA has been emerged into optimum filter designs. The characteristics of multi-objective, coded variables, and natural selection make GA different from other optimization techniques[2,3]. Filters designed by GA have the potential of obtaining near global optimum solution.

In this work, an effective design method for minimum phase digital FIR filters using GA is presented. This paper is organized as follows. Section 2 contains a brief review of basic GA. Section 3 describes the modified GA and its application to the design of minimum phase FIR digital filters. Section 4 presents the simulation results.

II. BASIC GENETIC ALGORITHM

The genetic algorithm is an artificial genetic system based on the process of natural selection and genetic operators. It is also a heuristic algorithm which tries to find the optimal results by decreasing the value of objective function (error function) continuously. A simplified GA cycle is shown in Fig.1.

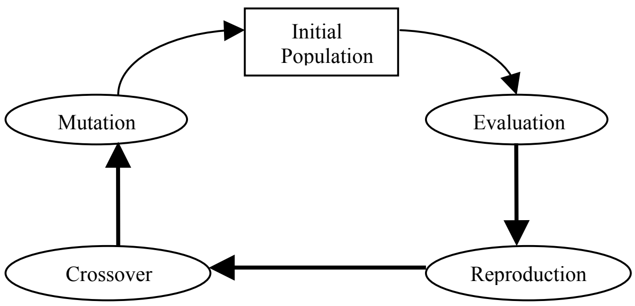


Figure 1. A simplified GA cycle

Initial population consist of a collection of chromosomes[2] and in practice it represents a set of solutions for the problem. The chromosome which produces the minimum error function value represents the best solution. The chromosomes which represent the better solutions are selected by the reproduction operator and then sent to the crossover operation. In this operation, two new chromosomes are produced from two chromosomes existing in the population. A common point in the selected chromosomes is randomly chosen and their corresponding digits are exchanged. Thus, new chromosomes which represent the new solutions are produced. The process in a simple crossover (single-point) operation is shown in Fig.2.

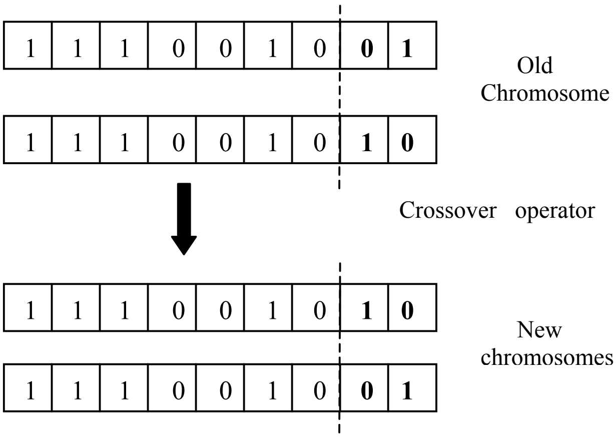


Figure 2. Crossover process

The next operator is mutation. Generally, over a period of several generations, the genes tend to become more and more homogenous. Therefore, many chromosomes can not continue to evolve before they reach their optimal state. In the mutation process, some bits of the chromosomes mutate randomly. Namely, certain digits will be altered from either ‘0’ to ‘1’ or ‘1’ to ‘0’ in binary encoding[4].

The GA used in this study has been written with MATLAB programming language and in addition to the operators mentioned above GA also contains ‘Elite’ operator. By means of Elite operator, the best solution is always kept. In the evaluation process, the solutions in the population are evaluated and a fitness value associated with each solution is calculated. These fitness values are used by the selection operator. Roulette Wheel method is employed for the selection process.

III. MODIFIED GENETIC ALGORITHM

In the basic Genetic Algorithm, to improve the fitness value of the chromosomes (represents a possible FIR filter) basic error functions are used. The chromosomes which have higher fitness values represent the better solutions. In the filter design the following error functions can be used : Mean Squared Error (MSE), Least Mean Squared Error (LMS), Minimax Error or Mean Absolute Error (MAE) [5,6]. However, from the simulation results, it is seen that the use of MSE in the passband and in the stopband regions and MAE in the transition band produces better results than other cases. The expressions of MSE and the MAE error functions are as follows,

$$MSE = \sum_f \left[\,|H_D(f)| - |H(f)|\, \right]^2 \tag{2}$$

$$MAE = \sum_f \left[\,|H_D(f) - H(f)|\, \right] \tag{3}$$

In conventional applications, only one of these error functions can be used as an error function. However, as we mentioned above to optimize the magnitude response and to provide the minimum phase we need to use both of these error functions, simultaneously. In order to realize this, in the GA used MSE error function is employed in the passband and in the stopband regions, and MAE error function is simultaneously used in the transition band during the design process. This is shown in Fig. 3,

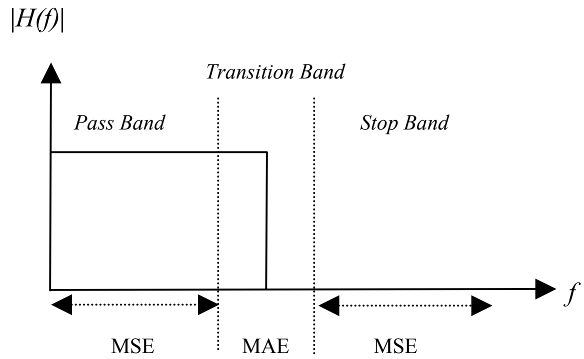


Figure 3. Objective function structure used

When the error functions are used directly as they are in Equation (2) or Equation (3) , it is seen that the magnitude response of the designed filter can not be efficiently optimized as expected and the minimum delay, namely minimum phase, can not be provided. It is assumed that the number of zeros that causes the non-minimum phase is q and the error function used is $e(f)$, then the objective function to be minimized which is able to provide the minimum phase condition can be defined as,

$$\psi(f) = e(f) + w.q \tag{4}$$

w = weight parameter

In expression (4), w has to be selected high enough. As the number of zeros that causes the non-minimum phase increases, the effect of these zeros on the error function will increase proportionally. Hence, by means of objective function the zeros which are located out of the unit circle are pulled into to the inside of the unit circle. When all the zeros are pulled inside the unit circle, the error function will be equal to the objective function since q=0.

The fitness evaluation function used in this section is given by Equation (5).

$$Fitness = \frac{1}{\psi(f)} \tag{5}$$

After several trials, it is seen that the most appropriate value for the parameter w in Equation (4) is 100. When the value of w is chosen too high, the value of the zero term becomes dominant on objective function and therefore, the GA might have difficulties with converging and finally could not reach the optimum solution. When the value of w is chosen too low, the influence of the zero term on the objective function becomes too small and hence the GA ignores this zero term and the designed filter might become non-minimum phase.

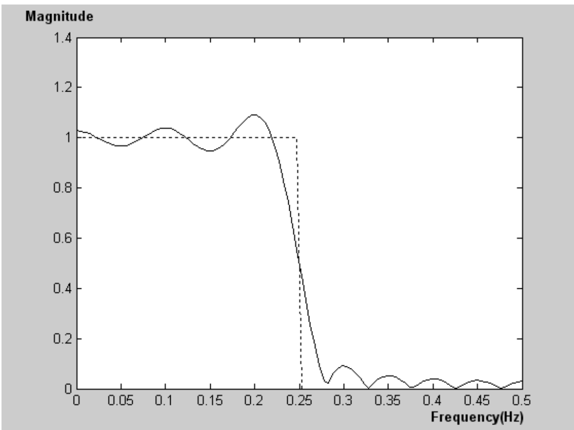
IV. SIMULATION RESULTS

The modified algorithm was applied to the 9th order minimum phase low pass FIR filter design problem. In the simulations, the sampling frequency was chosen $f_s = 1Hz$. The control parameters of GA used in this work are as the following:

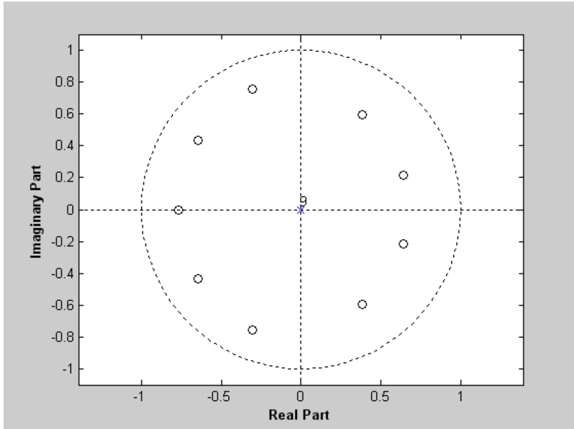
Generation Number	: 3.000	Population Size	: 100
Crossover Rate	: 0,6	Mutation Rate	: 0.01

As mentioned above, two modifications are made on the basic GA. In the first step, the objective function is improved such that it pulls the zeros which are located out of the unit circle into the inside of the circle. In the second step, the algorithm is modified such that it uses two different error functions to evaluate the designed filters.

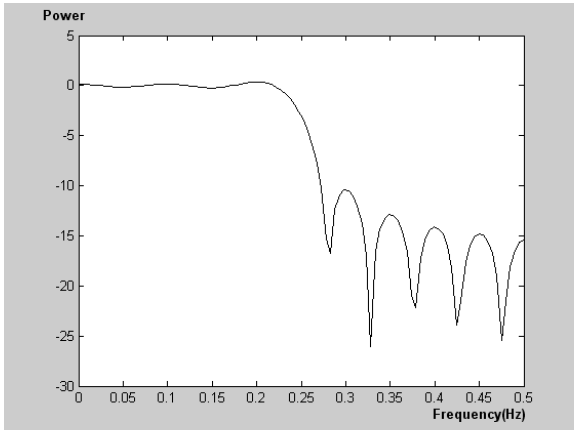
The simulation results obtained by using GA when MSE error function is used in passband, stopband and transition band regions (first modification) are given in Fig.4,



(a) Magnitude Response



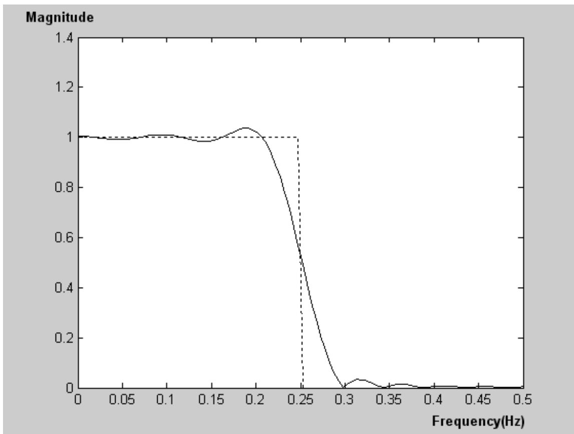
(b) Pole-Zero Diagram



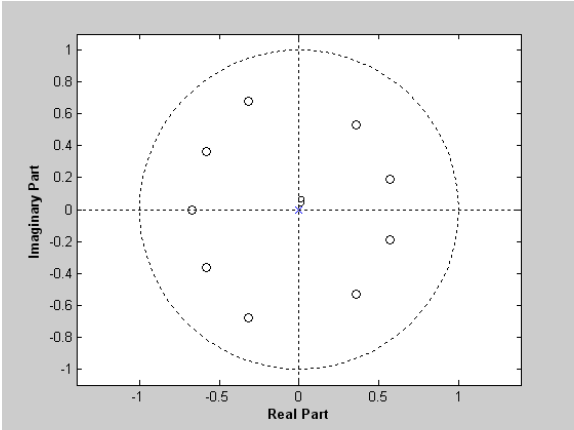
(c) Power Spectrum

Figure 4. The results obtained after the first step of modification

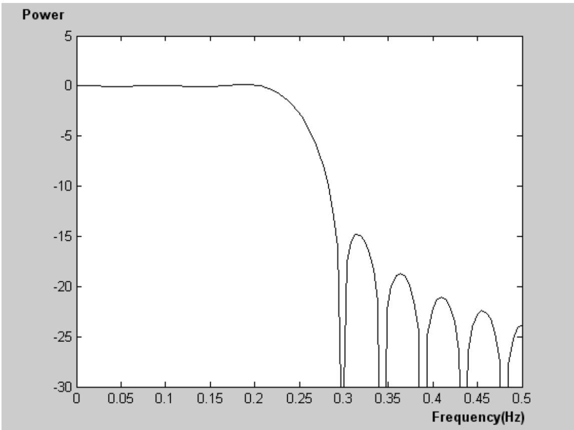
The simulation results obtained by using GA when MSE is used in passband, stopband and MAE in transition band regions (second modification) are given in Fig.5,



(a) Magnitude Response



(b) Pole-Zero Diagram



(c) Power Spectrum

Figure 5. The simulation results obtained after the second modification

V. CONCLUSION

As seen from the simulation results, the proposed method is able to design minimum phase filters, namely, all the zeros lie inside the unit circle. Besides, the desired responses can be obtained. The ripples in the pass band and in the stopband regions are attenuated succesfully. Filters with zeros inside the unit circle realize a given magnitude with the lowest possible delay, which is equivalent to the lowest possible phase variation.

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